Fermion vacuum energies in brane world models

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Abstract

The fermion representations and boundary conditions in five dimensional anti de Sitter space are described in detail. In each case the one loop effective action is calculated for massless fermions. The possibility of topological or Wilson loop symmetry breaking is discussed.

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Brane world models contain two types of fields, some restricted to four dimensional sheets and some living in the higher dimensional bulk [1]. The quantum properties of those fields that exist in the bulk is a rich subject. In this paper we focus on a simple example, namely massless fermions in five dimensions, and consider the effective action and the possibility of topological symmetry breaking [2–5].

One of the issues which arises is the contribution that vacuum fluctuations make towards the stability of two parallel branes. This is important for the Randall-Sundrum scenario, which relates the mass hierarchy problem to brane separations in anti-de Sitter space [6]. The effects of vacuum fluctuations have already been considered for scalar and fermion fields [7–12]. However, we feel the need to clarify some statements that have been made about the fermion boundary conditions in these models and to calculate the effective action for a variety of boundary conditions.

The theory we consider is five dimensional. The fifth dimension is taken to be an orbifold S_1/Z_2, where the circle runs from $y = -a$ to $y = a$ and the Z_2 acts by $y \to -y$. The space is equivalent to a five dimensional spacetime with two four dimensional branes making up the boundary.

The choice of fermion representations in five dimensions is as wide as it is in four. To start with, we might require that the Lagrangian be invariant under the full five dimensional Lorentz group. The Lorentz symmetry in this case would then only be broken by the presence of the brane worlds. The fermions transform by a matrix S, related to a set of gamma matrices Γ^a. For the Lorentz transformation $y \to -y$,

$$S^{-1}\Gamma^5 S = -\Gamma^5 \tag{1}$$

$$S^{-1}\Gamma^\mu S = \Gamma^\mu \tag{2}$$

The smallest representation of the gamma matrices which can satisfy these relations has eight component spinors. The existence of a matrix Γ^6 which anticommutes with the other gamma matrices allows $S = i\Gamma^5\Gamma^6$. The benefits of using eight component spinors have also been emphasised in discussions of fermion boundary value problems [13].

The eight dimensional representation can be reduced to a real eight dimensional Majorana representation, where $\psi = C\psi^*$ and C satisfies $C^{-1}\Gamma^a C = -\Gamma^{a*}$. These are the fermion representations which arise from the reduction of supersymmetric theories [14]. In the supersymmetry literature, the eight dimensional fermions are usually regarded as a pair of four dimensional fermions related by a symplectic transformation [15].

If, instead of the full Lorentz group, we require only symmetry under proper Lorentz transformations, then the representation reduces to four dimensional Weyl representations. These are the representations that have been considered hitherto in the context of the Randall-Sundrum scenario [16]. A rule for transforming spinors under the transformation $y \to -y$ is still required. For this purpose, we can allow the massless Dirac equation to transform as a pseudoscalar,

$$S^{-1}\Gamma^5 S = \Gamma^5 \tag{3}$$

$$S^{-1}\Gamma^\mu S = -\Gamma^\mu \tag{4}$$

The solution is $S = i\Gamma^5 = \gamma^5$, where $\Gamma^\mu = \gamma^\mu$ are the usual gamma matrices in four dimensions. We will refer to these fermions as 'five dimensional Weyl fermions'. They have the useful property that they induce a chiral particle theory on the branes [16].

The boundary conditions used in this paper are chosen for consistency with orbifold reductions of the fifth dimension. The fermions carry a representation of the Z_2 symmetry, hence $\psi(y) = \pm S\psi(-y)$. Therefore, if

$$P_\pm = \tfrac{1}{2}(1 \pm S), \tag{5}$$

we must impose one of the two equivalent conditions $P_\pm\psi = 0$ at $y = 0$.

The fact that y lies on a circle would normally imply that $\psi(a) = \psi(-a)$. However, if there is a symmetry it is possible to make the identification up to a symmetry transformation. The possibility $\psi(a) = -\psi(a)$ has already been used as a mechanism for breaking supersymmetry, [17]. More general possibilities which would allow gauge symmetry breaking are discussed later, but for the moment we have two inequivalent boundary conditions

$$y = 0: \quad P_-\psi = 0, \quad y = a : P_-\psi = 0 \tag{6}$$

$$y = 0: \quad P_+\psi = 0, \quad y = a : P_-\psi = 0 \tag{7}$$

The first set might be regarded as untwisted and the second set twisted. The twisted case has been considered in flat five dimensional models by Antoniadis et al. [18,19]. For Weyl fermions, the untwisted boudary condition agrees with the boundary conditions used by Grossman et al. [16] and gives results similar to Garriga et al. [9]. We will give results for both Dirac and Weyl fermions and for both twisted and untwisted cases.

We will take the metric

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2. \tag{8}$$

Anti de Sitter space corresponds to $\sigma = \kappa y$, κ being a constant. This metric is conformally flat,

$$ds^2 = e^{-2\sigma}(\eta_{\mu\nu}dx^\mu dx^\nu + d\tau^2) \tag{9}$$

where $0 \le \tau \le \beta$, and

$$\beta[\sigma] = \int_0^a e^\sigma dy. \tag{10}$$

For anti de Sitter space, $\beta = \kappa^{-1}(e^{\kappa a} - 1)$.

When boundaries are present it is convenient to regard the Dirac operator D mapping one set of fermions into an image set. The adjoint mapping is denoted by D^*. The one loop contribution to the effective action is then

$$W = -\tfrac{1}{2}\log\det(D^*D). \tag{11}$$

The boundary conditions on the image fermions can be determined by the existence of D^*. If $P_-\psi = 0$, this requires that the normal derivative of $P_+\psi$ should vanish.

In the massless case, $D = -D^* = i\Gamma^j\nabla_j$ where j runs from 1 to 5. The conformal transformation properties of the massless operator imply $D = e^{3\sigma}D_0e^{-2\sigma}$, where D_0 is the Dirac operator in the strip of flat space $0 \le \tau \le \beta$. The boundary conditions are also conformally invariant. We can relate the effective action to the result in flat space by

$$W = W_0 + C[\sigma] \tag{12}$$

where $W_0 = -\frac{1}{2}\log\det(D_0^* D_0)$ and $C[\sigma]$ is a correction term [20]. We shall discuss the significance of this term later.

For untwisted Dirac fields we have $P_-\psi = 0$ and $\partial(P_+\psi)/\partial\tau = 0$ on either boundary. The eigenvalues of $D_0^* D_0 = -\nabla^2$ are then $k^2 + m_n^2$, with $m_n = \pi n/\beta$, where $n = 0, 1, 2 \ldots$. The degeneracy $g = 8$ for each value of k. Twisted Dirac fields have similar eigenvalues with $m_n = (n + \frac{1}{2})\pi/\beta$. For Weyl fermions, the eigenfunctions and eigenvalues are unchanged, but now the degeneracy factors are $g = 4$ rather than $g = 8$.

The logarithms can be evaluated using ζ-function regularisation [20,21]. For untwisted fields,

$$W_0 = \int d^4x \, \frac{3g}{128\pi^2}\zeta_R(5)\beta^{-4} \tag{13}$$

where ζ_R is the Riemann ζ-function. For twisted fields,

$$W_0 = -\int d^4x \, \frac{3g}{128\pi^2}\frac{15}{16}\zeta_R(5)\beta^{-4}. \tag{14}$$

The results depend on the separation of the branes only through β given in equation (10). The untwisted Weyl case gives the same result as that obtained by Garriga et. al. [9].

In this particular problem there is no dependence on the renormalisation scale. The same result can be obtained by dimensional regularisation where the absence of pole terms also indicates no dependence on the renormalisation scale. The situation changes when the branes are curved, and renormalisation scale dependent curvature terms arise [8].

The quantity W_0 is also the one loop correction to the effective action of an infinite set of particles in four dimensions with mass m_n. The difference between W and W_0, namely the cocycle function $C[\sigma]$, can be regarded as an anomaly in this reduction. Such anomalies have been recognised by Frolov et al. [22,23]. In general, $C[\sigma]$ will depend on the geometry of the branes, but in the present context the anomaly only contributes a constant term to the matter Lagrangians $\mathcal{L}_v$ and $\mathcal{L}_h$ on the 'visible' and 'invisible' branes.

In the Randall-Sundrum metric (8), the classical action reduces to

$$S = -\int d^4x e^{-4\kappa a}\left(\mathcal{L}_v - \frac{3\kappa}{4\pi G_5}\right) - \int d^4x \left(\mathcal{L}_h + \frac{3\kappa}{4\pi G_5}\right), \tag{15}$$

where G_5 is the gravitational constant in five dimensions. Junction conditions on the metric imply that both terms vanish in the vacuum. Adding the correction W for untwisted fermions gives an effective action which now has a minimum for a particular separation a. However, the values obtained cannot give the correct mass hierarchy ($\kappa a > 30$) without a considerable degree of fine tuning.

We can also derive the vacuum energy density from the effective action quite simply,

$$T_{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta W}{\delta g^{\mu\nu}} = e^{5\sigma}\frac{dW}{d\beta}g_{\mu\nu}. \tag{16}$$

For the untwisted anti-deSitter case, $\sigma = \kappa y$, the energy density is

$$\frac{3g}{32\pi^2}\zeta_R(5)\kappa^4(e^{\kappa(a-y)} - e^{-\kappa y})^{-5}. \tag{17}$$

Since this is strongly concentrated near the visible brane, the back reaction of this energy modifies the junction conditions, resulting in a value for a in agreement with the minimum of the effective action.

If there is a gauge symmetry, the possibility of topological or Wilson loop symmetry breaking arises. The boundary conditions can be generalised by inserting gauge transformations U_h and U_v on the two branes, so that now

$$P_- = \tfrac{1}{2}(1 - SU), \tag{18}$$

where $U = U_h$ or $U = U_v$. The condition $P_-^2 = P_-$ requires $U_h^2 = U_v^2 = I$. The symmetry is broken, leaving the centraliser of U_h and U_v, which preserves the boundary conditions, as the residual symmetry group.

For a simple, non-trivial, example, consider the group $U(2)$ with $U_h = \sigma_3$, the Pauli matrix. If $U_v = I$, the residual symmetry group is $U(1) \times U(1)$ and the fermions decompose into one twisted and one untwisted fermion. The combined one loop correction is therefore $\frac{1}{16}W_0$, where W_0 is the untwisted result (13). If $U_v = \pm\sigma_3$, the residual symmetry group is the same but the fermions are both twisted or both untwisted, giving a correction $2W_0$ or $-\frac{15}{8}W_0$. The final case is represented by $U_v = \sigma_1$, and the residual symmetry group is $U(1)$. The eigenvalues are now of the form $k^2 + m_n^2$, with $m_n = (n \pm \frac{1}{4})\pi/\beta$. The effective action can be calculated as before, and takes the value $-\frac{15}{256}W_0$.

For massless fermions, the separation between the two branes is only stable when the correction to the action is positive. Clearly, there is an interplay in this scenario between supersymmetry breaking, gauge symmetry breaking and the mass hierarchy problem. We are presently extending these results to massive fermions so that we can investigate the consequences in low energy superstring models.

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